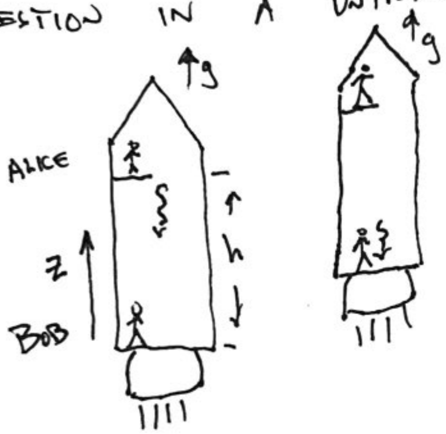
Q: WHAT IS THE CLOCK TIME $\Delta\tau_B$ IN COMPARISON TO $\Delta\tau_A$?

BY THE EINSTEIN EQUIVALENCE PRINCIPLE THIS IS THE SAME QUESTION IN A UNIFORMLY ACCELERATING FRAME:



TIME FOR ALICE TO RELEASE LIGHT SIGNALS

$$t = 0$$
$$t_1 = \Delta\tau_A$$
$$t_2 = 2\Delta\tau_A$$
$$\vdots$$

BOB RECEIVES THESE AT

$$t' = t_1 = \frac{h}{c}$$
$$t'_1 = t_1 + \Delta\tau_B$$
$$t'_2 = t_1 + 2\Delta\tau_B$$
$$\vdots$$

POSITION OF BOB

$$z_B = \frac{1}{2} g (t)^2$$

POSITION OF ALICE

$$z_A = h + \frac{1}{2} g (t)^2$$

→ LIGHT TRAVELS FROM ALICE TO BOB

- 1ST PULSE OF LIGHT:

$$z_A(0) - z_B(t_1) = c t_1 \qquad (1)$$

- NEXT PULSE OF LIGHT:

$$z_A(\Delta\tau_A) - z_B(t_1 + \Delta\tau_B) = c\left(t_1 + \Delta\tau_B - \Delta\tau_A\right) \qquad (2)$$

EXPANDING Z'S AND DROPPING $\Delta\tau_A^2$ AND $\Delta\tau_B^2$ TERMS (WE ASSUME THESE ARE SMALL)

FROM (1)

$$h - \frac{1}{2} g t_1^2 = c t_1$$

FROM (2)

$$h + \cancel{\frac{1}{2} g \Delta\tau_A^2} - \frac{1}{2} g \left(t_1^2 + 2 t_1 \Delta\tau_B + \cancel{\Delta\tau_B^2} \right) = c \left(t_1 + \Delta\tau_B - \Delta\tau_A \right)$$

SUBTRACTING THESE TWO EQN'S

$$g t_1 \Delta\tau_B = c \left(\Delta\tau_A - \Delta\tau_B \right)$$

BUT $t_1 = \frac{h}{c}$ SO

$$\frac{gh}{c} \Delta\tau_B = c \left(\Delta\tau_A - \Delta\tau_B \right)$$

OR

$$\Delta\tau_B = \frac{\Delta\tau_A}{1 + \frac{gh}{c^2}}$$

SINCE $\frac{gh}{c^2}$ IS SMALL, $\Delta\tau_B \simeq \Delta\tau_A \left(1 - \frac{gh}{c^2} \right)$

(USING $(1+x)^n \simeq 1 + nx$ FOR $x < 1$)